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Use of a miniature wind turbine with rectangular blades to demonstrate and confirm the existence of optimal angles of inclination of turbine blades at which peak power is produced

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ABSTRACT: The testing of a miniature wind turbine equipped with three rectangular blades demonstrated that power produced by that turbine varied with the speed of the wind and with the angle of inclination of the turbine blades to the wind direction. Tested angles of inclinations varied from 0˚ to 90˚ and each angle was tested using wind speeds that ranged from 0.1 m/s to 4.5 m/s. It was found that peak power produced by the turbine always occurred in the same narrow range of angles of inclination of the turbine blades. These findings support and confirm similar results obtained in a previous investigation that had used a different turbine and a different fan. Furthermore, the current study suggests that, when it comes to the determination of the coefficient of performance of a miniature wind turbine, the so-called *interference factor* can be accurately described using a simple mathematical function that is related to the angle of inclination of the turbine blades. For the data collected in this study, it was shown that choosing the sine of the angle of inclination of the turbine blade with the vertical, such a function worked very well.

INTRODUCTION

A graph that indicates how much power (in watts, kilowatts or megawatts) a wind turbine will produce as a function of wind speed is called the *power curve* of that turbine. In a typical power curve, power is presented along the vertical axis (ordinates) and wind speed on the horizontal axis (abscissas). A common issue that arises with power curves relates to whether or not such a chart can be believed. That is, can the buyer of a wind turbine depend upon what is indicated on the power curve produced by the manufacturer of a wind turbine? The answer to this question has been controversial in the USA for about forty years [1]. And the answer depends upon the size of the wind turbine.

Wind turbines are classified using different criteria. If power produced is used as a criterion, then, by convention, there are three categories of wind turbines: large, medium or midsised, and small. This is based on the maximum (or peak) power that they can produce. A wind turbine is said to be large if the peak power it can produce is larger than 1,000 kW (or 1 MW); it is said to be midsised if the peak power it can produce is somewhere between 100 kW and 1,000 kW (or 1 MW); and it is classified as small if the peak power that it can produce is less than 100 kW [2].

Generally, for large wind turbines, it has been determined that power curves provided by manufacturers are very reliable. Indeed, for this class of turbine, performance often exceeds that indicated on the power curve [1]. However, such is not the case for small turbines. In this category, some designs underperform; others perform as indicated; and still others perform beyond the standards that power curves indicate [1-3].

Attempting to introduce wind energy into the engineering curriculum is a challenge, if such an introduction must include hands-on laboratory work. Consider the simple observation that the blades of modern wind turbines are essentially airfoils that are tapered and twisted, which causes the angle of inclination of the turbine to the vertical to vary along its length [4-7]. How are students to gain hands-on experience with this important aspect of wind turbines? A major challenge is that wind turbines are very expensive. Their costs vary from tens of thousands to millions of dollars (USA) [8]. For purposes of undergraduate instruction, therefore, the use of commercial wind turbines to investigate, say, the roles of curvature and angle of inclination on the power produced is totally unrealistic. An effective way to help undergraduates learn the aerodynamics of turbine blades is through the use of miniature wind turbines that individual students and their fellow classmates can design, build and test [9].

In earlier investigations, a series of tests were conducted using miniature turbines equipped with rectangular blades. The purpose of those investigations was to explore the role of the angle of inclination of the turbine blades in the production of voltage. In one such test, 13 different inclinations of the turbine blades were tested through a variety of wind speeds generated by a household fan. Results indicated the existence of optimal angles of inclination at which a given turbine design produced its maximum power. Such peak voltages varied with the distance between the fan and the face of the turbine blades. However, all peak voltages were found to occur only at angles of inclinations that were between 20 and 30 degrees relative to the vertical [10].

A question that arose from these findings was whether or not such conclusions could be duplicated, if a different turbine were designed and a different fan was used to produce flow patterns and wind speeds that were different from those already tested. The purpose of this article is to report the results of an experiment that demonstrates and confirms that, indeed, optimal angles do exist of inclinations at which peak power is produced by a miniature wind turbine.

The article is organised in the following manner: first, the experimental setup is described. Then, the procedure used to collect data is presented. Next, collected data are presented in tabular and graphical forms that allow the existence of the optimal angles to be demonstrated. Finally, salient results are summarised about the existence of optimal angles of inclination of turbine blades at which peak power is produced by a miniature wind turbine.

EXPERIMENTAL SETUP

An experiment was designed in order to collect data from a miniature wind turbine to investigate the existence of optimal angles of inclination of turbine blades at which maximum (or peak) power is produced by the turbine. Engineering students had constructed miniature wind turbines for various projects using ordinary items [9][10]. One of those wind turbines was chosen for this investigation. The materials and design of the blades of that turbine were changed from the propeller blades of model airplanes to rectangular blades made from balsa wood.

The base and tower of the miniature wind turbine consisted of PVC piping (Figure 1a). A small DC motor was housed within another piece of PVC pipe located at the top of the tower (Figure 1b). Wires connected to that motor ran through the PVC piping towards the base of the turbine and were connected to a Fluke 8050A Digital Multimeter, which was used to measure the voltage and current outputs generated from the DC motor (Figure 1c). The measured voltage and current were used to calculate the power output from the turbine. Balsa wood was used to construct three rectangular blades with dimensions 6 in x 2 in (15.24 cm x 5.08 cm). Amaco WireForm soft metal rods were glued to the balsa wood using Gorilla Glue to attach the blades to the rotor of the turbine. A Galaxy, Model 4733, three-speed box fan was used as the wind source (Figure 1d). The three speed settings of the fan were designated by the manufacturer as 1 (low), 2 (medium) and 3 (high). The experimental station was set on top of a long and sturdy table. The length of the table made it possible to vary the distance between the fan and the turbine; this allowed for the testing of the set up through a wide variety of wind speeds, thereby, collecting an abundant amount of data (Figure 1e).

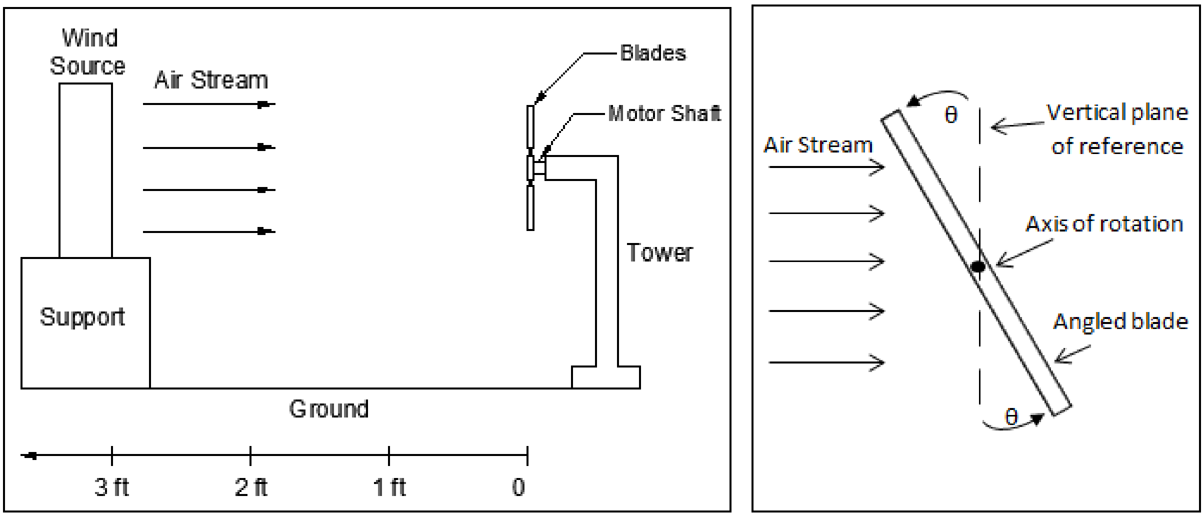
a) b) c) d) e)

Figure 1: The equipment used for the testing of the wind turbine: a) the three-bladed turbine; b) the DC motor; c) the multimeter connection; d) the three-speed box fan; and e) side view of the experimental station.

EXPERIMENTAL PROCEDURE

The experimental station was set up as shown in Figure 2a to provide an easily reproducible method of testing how the amount of power produced by a wind turbine varied with the wind speed and the angle of inclination of the blades. The base of the turbine was secured to the table using tape; and the fan was positioned directly in front of the turbine, making sure that the turbine blades were fully immersed in the air flow from the fan. In order to vary the wind speed, the fan was moved away from the turbine from 1ft to19ft (0.30 to 5.8 m), using 1-foot (0.3 m) increments. This made a total of 19 different fan positions. The voltage and current produced under the low, medium and high fan settings were measured at each position of the fan, respectively. The front faces of the fan and the turbine were set upward in the vertical plane. Thus, air blown by the fan moved in the horizontal direction and, when strong enough, it caused the turbine blades to spin about the horizontal axis of the turbine.

The angle of inclination of the blades was successively set to 0˚, 5˚, 10˚, 15˚, 20˚, 25˚, 30˚, 35˚, 40˚, 45˚, 50˚, 55˚, 60˚, 65˚, 70˚, 75˚, 80˚, 85˚ and 90˚. These angles were measured relative to the vertical plane as shown in Figure 2b. Therefore, an angle of $\theta = 0^\circ$ corresponded to a vertical blade that was perpendicular to the air flow. Likewise, an angle of θ = 90˚ corresponded to a blade that was positioned parallel to the wind flow. At each such angle, tests were run using each of the three speed settings of the fan; and the voltage and current were measured at each fixed combination of fan speed and angle of inclination. Therefore, for each such combination, 19 different wind speeds were tested, each corresponding to a fixed distance between the fan and the turbine.



a) b)

Figure 2: a) Side view of the experimental set up used to collect data from a miniature wind turbine; b) sketch defining the angle of inclination of the turbine blades relative to the vertical (shown from the top view).

All specified angles were tested at all specified distances (wind speeds). It was observed that only angles between 10˚ and 80˚ were able to produce amounts of voltage and current that were measurable by the Fluke 8050A Digital Multimeter. To start collecting the data, the fan was placed 1 foot (0.3 m) away from the turbine, the blade angle was set, and the speed was set to high. The current and voltage measurements were recorded for that combination of parameters. The speed was reduced to medium and the new current and voltage measurements were recorded. The speed was then set to low and the respective voltage and current measurements were recorded.

Next, the fan speed was increased back to high and the fan itself was moved further away from the turbine by 1 foot (0.3 m). This whole four-step process was repeated over and over again, until a critical distance was reached at which, and beyond which, neither voltage nor current measurements could be registered by the digital multimeter. From this and longer distances, it was observed that the intensity of the wind speed that reached the turbine was too weak to make the turbine blades spin.

EXPERIMENTAL RESULTS

When the distance between the fan and the turbine was fixed and the power produced by the turbine at that fan location was plotted versus the angles of inclination of the turbine blades, the resulting plot yielded a curve that first increased with increasing angle of inclination until a peak was reached. However, as the angles increased further, the power produced by the turbine decreased continuously. Such plots were obtained for each tested fan location that yielded non-zero data. All such curves displayed the same pattern and similar shapes.

The general shape of the plot of the power versus the angle of inclination is shown in Figure 3. A hypothetical line has been added to the plot shown in Figure 3, in order to represent the general form of the curve that would have been expected, if power had been measured at small angles that span the interval between zero and the first angle at which the data were actually collected.

Distance and speed settings were used as parameters to discover the relationship between the power produced by the turbine and the angle of inclination of the turbine blades. In all cases tested, the power produced increased as the angle increased from 0˚ and reached a peak somewhere between 15˚ and 25˚. The data collected indicated that the maximum power a miniature turbine with rectangular blades can produce occurred when the angle of inclination was set to 20˚±5˚. Once the angle was set above 25˚, the power started to decrease continuously. This pattern held true regardless of the speed setting of the fan and regardless of the distance between the fan and the turbine (wind speed).

When all the angles of inclination at which the blades of the miniature turbine produced maximum power were identified, it became clear that the angle at which peak power was produced most frequently was 20˚. There were a total of 51 angles at which maximum power was produced. Only 1 of them occurred at 10˚; only 1 occurred at 15˚; 41 of them occurred at 20˚ (80.4%); and 8 at 25˚ (15.7 %). It can be seen that, by far, the maximum power produced by the miniature turbine was produced when the angle of inclination was 20˚.

While a few other angles produced maximum power, the number of such instances (angles 10/51, 19.6%) is significantly smaller than the number of instances at the 20˚ angle. The detailed counts and percentages for each individual speed setting of the fan are shown in Table 1. It is noteworthy that only one instance resulted in the maximum power being produced at an angle outside the 15˚-25˚ range and that measurement was recorded at 10˚ for the medium speed setting.

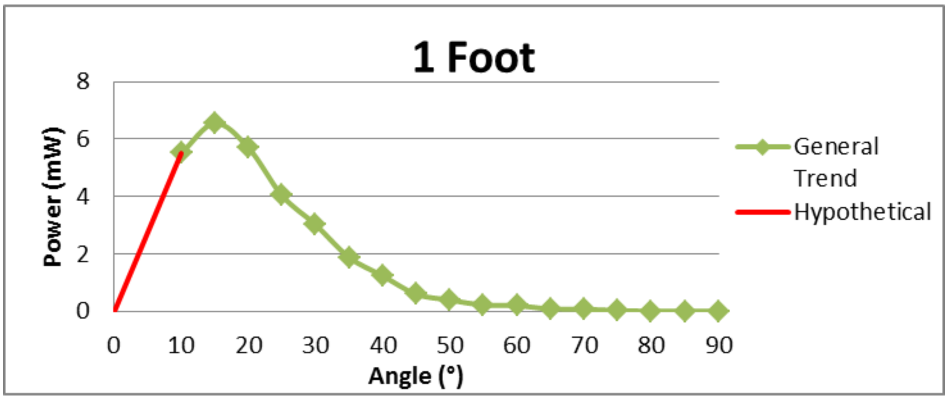


Figure 3: The general trend of the plots of power versus angle of inclination indicated by the collected data.

Table 1: The number of instances that produced the maximum power and the corresponding angle of the blades.

Angle of maximum power	At low speed setting	At medium speed setting	At high speed setting
15˚	0	0	1 (5.3%)
20˚	15 (100%)	13 (76%)	13 (68.4%)
25˚	0	3 (18%)	5 (26.3%)
Other Angles	0	1 (6%)	0
Total	15	17	19

Based on the data collected, the angle at which the maximum power was produced generally increased slightly as the distance between the turbine and the fan increased, especially for the medium and high speed measurements. The five instances of the high-speed setting which resulted in the 25˚ angle producing the most power occurred for the five largest distances between the fan and the turbine. Similarly, two of the three instances for the medium speed which were at 25˚ were recorded at the two largest distances between the fan and the turbine.

The data demonstrate that the angle of 20˚±5˚ produced the most power for the miniature wind turbine that was tested. This angle increased slightly as the distance between the fan and the turbine increased. The increase in distance was equivalent to reducing the wind speed. Therefore, at large wind speeds, when the turbine was close to the fan, the angle that produced the most power was very close to 20˚. As the wind speeds decreased, the angle that produced the most power tended to increase slightly but was never greater than 25˚. Consequently, the optimal angle of inclination varied slightly depending on the wind speed, but was generally between 20˚and 25˚.

POWER PRODUCED VERSUS ANGLE OF INCLINATION OF THE TURBINE BLADES

Plots of the data that show power versus the angle of inclination of the turbine blades when the fan was 1 foot (0.3 m) away from the turbine are presented in Figure 4. For each speed setting of the fan, these plots show that the maximum amount of power produced occurred when the angle of the blades was set to 20˚. All angles at which a significant amount of power was produced by the turbine were between 10˚ and 30˚. Angles greater than 30˚ and angles greater than 45˚ produced only about 30% and 10% of the maximum possible amount of power, respectively.

For a fixed fan location, the amount of power decreased greatly with increasing angle of inclination after peak power was achieved. Therefore, in addition to wind speed, the angle of inclination of the turbine blades is a major factor in determining the amount of power that a wind turbine with rectangular blades will be able to produce. A similar general trend was observed at distances greater than 1 foot (0.3 m). This is illustrated in Figure 5 by using three other sample distances.

In Figures 5a, 5b and 5c, it can be seen that power decreased as the distance increased between the fan and the turbine. This means that, as the speed of the wind hitting the blades decreases, so did the amount of power produced by the turbine. Comparing the general trends displayed by the data in Figures 5a, 5b and 5c, it can be seen that they are relatively the same. In each case, Figures 5a, 5b and 5c start to rise as the angle is increased until they reach a maximum near the 20˚ angle. After the 20˚ angle, they start to decrease and do so very rapidly. A slight increase appears around the 40˚ angle area on all three plots. At the 40˚ angle, however, the turbine was only producing about 10% of the power that was produced at smaller angles at the same distance. This was true for all the data collected. After the 40˚ angle, the power decreases at a slower rate, and the amount of power produced began to approach zero.

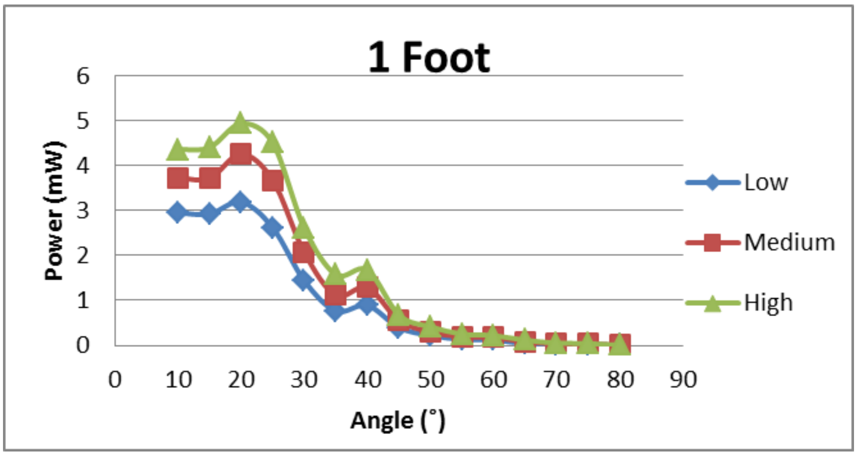
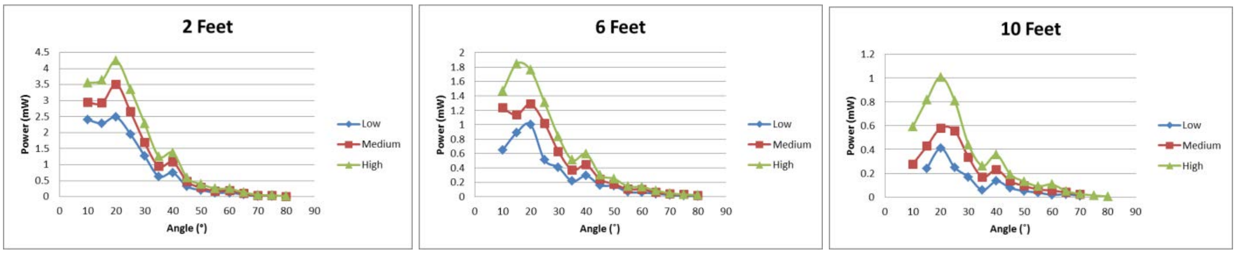


Figure 4: Power versus angle of inclination of the blades when the fan was at a distance of 1 foot (0.3 m) from the turbine.

The sample plots shown in Figures 5a, 5b and 5c illustrate what was observed in all cases: 1) the relationship between power produced by the turbine and the angle of inclination of turbine blades showed a similar general trend, regardless of the distance between the fan and the turbine; and 2) the optimal angles at which maximum power was produced by the miniature wind turbine were found around 20˚±5˚, regardless of the wind speed.



a) b) c)

Figure 5: Samples of the plots of power versus angle of inclination of the blade for different positions of the fan: a) 2 feet (0.6 m) between the fan and the turbine; b) 6 feet (1.8 m) between the fan and the turbine; c) 10 feet (3 m) between the fan and the turbine.

OPTIMAL ANGLES OF INCLINATION AND THE IDEAL (BETZ) EFFICIENCY

It is customary to use a one-dimensional analysis of a wind turbine to investigate the power extracted from the wind. The ratio between the ideal power produced by the turbine and the Kinetic Energy Flux (KEF) of the fluid stream is used to define the efficiency of the turbine. It is given by Equation (1), where *a* is called the interference factor [5-7].

$$\eta = \frac{P_{ideal}}{KEF} = 4a(1-a)^2 \tag{1}$$

Experiments indicate that the amount of kinetic energy captured by turbine blades from a wind blowing over them depends on the angle of inclination of the blades to the wind direction. This is to be expected, because that angle influences the interaction between the wind and the turbine blades. Assuming that the interference factor could be captured by using a simple function of the angle of inclination of the turbine blades, it was set a = sin(θ) in Equation (1), where θ is the angle of inclination of the turbine blade relative to the vertical. Then, the expression for the ideal efficiency shown in Equation (1) becomes:

$$\eta = 4\sin\theta - 8\sin^2\theta + 4\sin^3\theta \tag{2}$$

A plot of the ideal efficiency shown in Equation (2) as a function of the angle of inclination of the turbine blades is shown in Figure 6. It is well known that the maximum efficiency, the so-called Betz limit, is 59.3% and that it occurs when the interference factor a = 1/3 [5-7]. Setting sin(θ) = 1/3, yields an optimal angle of 19.5˚, which is very close to the optimal angles of 20˚±5˚ that were obtained experimentally using the data collected in this study.

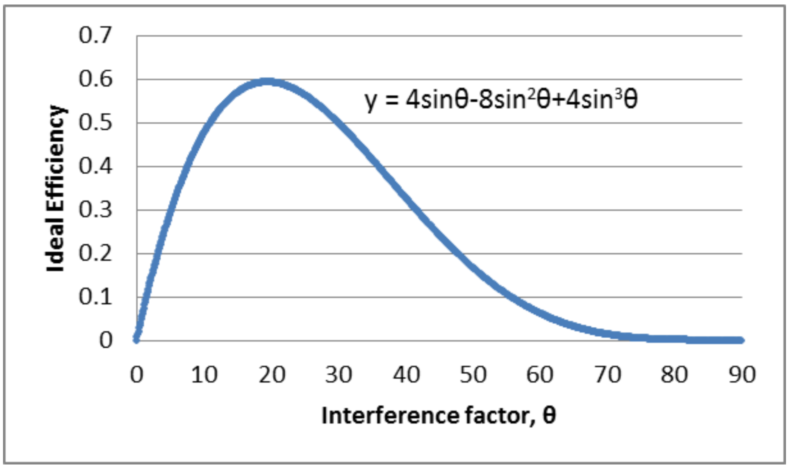


Figure 6: Variation of the ideal efficiency of the turbine with θ, the angle of inclination of turbine blades, Equation (2).

CONCLUSIONS

The testing of a miniature wind turbine equipped with three blades demonstrated that power produced by that turbine varied with the speed of the wind and with the angle of inclination of the turbine blades to the wind direction. Tested angles of inclinations varied from 0˚ to 90˚ and each angle was tested using wind speeds that ranged from 0.1 m/s to 4.5 m/s. It was found that peak power produced by the turbine always occurred over the same narrow range of angles of inclination of the turbine blades. These findings support and confirm similar results that were obtained in a previous investigation that had used a different turbine and a different fan [10].

Furthermore, the current study suggests that, when it comes to the determination of the coefficient of performance of a miniature wind turbine, the so-called interference factor can be accurately described using a simple mathematical function that is related to the angle of inclination of the turbine blades. For the data collected in this study, it is shown that choosing the sine of the angle of inclination of the turbine blade with the vertical as such a function worked very well.

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